Machine learning based surrogate modelling 2: Case FEM models

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What kind of a surrogate model for a periodically excited device?

The goal is to develop a "black box” surrogate model of a structure/device which is self-taught on the basis of measurements.

In this study a FEM model is used to develop and test the ideas and the surrogate model. A FEM model offers a familiar and accurate environment for the development work.

A harmonic oscillator …

Let’s start with a harmonic oscillator. It is the most simple vibrating system. Its equation of motion is

$$m\ddot{u}+c\dot{u}+ku=f(t)$$

m mass, c viscous damping coefficient, k spring constant, u(t) displacement from rest, and f(t) excitation force.

Its solution of the equation of motion in a steady state is

$$u(t)=U_c\cos(\omega t)+U_s\sin(\omega t)$$

When the excitation force is of same form there is an analytical solution. If the force is periodic, the excitation can be presented as a Fourier series.

$$f(t)=\sum\left(F_{cj}\cos(\omega_j t)+F_{sj}\sin(\omega_j t)\right)$$

… A harmonic oscillator

Because of the cosine and sine terms the equations double in the frequency domain. The equation and solution for an angular frequency is

$$\begin{bmatrix} (k - m\omega_j^2) & c\omega_j \\ -c\omega_j & (k - m\omega_j^2) \end{bmatrix} \begin{Bmatrix} U_{cj} \\ U_{sj} \end{Bmatrix} = \begin{Bmatrix} F_{cj} \\ F_{sj} \end{Bmatrix}$$

$$\begin{Bmatrix} U_{cj} \\ U_{sj} \end{Bmatrix} = \frac{1}{(k - m\omega_j^2)^2 + (c\omega_j)^2} \begin{bmatrix} (k - m\omega_j^2) & -c\omega_j \\ c\omega_j & (k - m\omega_j^2) \end{bmatrix} \begin{Bmatrix} F_{cj} \\ F_{sj} \end{Bmatrix}$$

In time domain the solution is

$$u(t) = \sum \left(U_{cj}\cos(\omega_j t) + U_{sj}\sin(\omega_j t) \right)$$

A FEM model with lots of degrees of freedom…

A FEM model with is defined by a mass matrix, a damping matrix, and a stiffness matrix (M, C, K).

$$\boldsymbol{M}\,\ddot{\mathbf{u}}(t)+\boldsymbol{C}\,\dot{\mathbf{u}}(t)+\boldsymbol{K}\,\boldsymbol{u}(t)=\boldsymbol{f}(t)$$

The solution is similar to the single degree of freedom harmonic oscillator. In the steady state the solution is

$$\boldsymbol{u}_j(t)=\boldsymbol{u}_{jc}\cos(\omega_j t)+\boldsymbol{u}_{js}\sin(\omega_j t)$$

The excitation is assumed to be of the form

$$\boldsymbol{f}(t)=\sum \boldsymbol{f}_j \qquad \boldsymbol{f}_j=\boldsymbol{f}_{cj}\cos(\omega_j t)+\boldsymbol{f}_{sj}\sin(\omega_j t)$$

…A FEM model with lots of degrees of freedom

In the frequency domain the system matrices double because of cosine and sine terms. The equation of motion in matrix form is

$$\begin{bmatrix} (\boldsymbol{K}-\boldsymbol{M}\omega_j^2) & \boldsymbol{C}\omega_j \\ -\boldsymbol{C}\omega_j & (\boldsymbol{K}-m\omega_j^2) \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_{cj} \\ \boldsymbol{u}_{sj} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_{cj} \\ \boldsymbol{f}_{sj} \end{Bmatrix} \quad \Leftrightarrow \quad \boldsymbol{K}_j \boldsymbol{u}_j = \boldsymbol{f}_j$$

Its formal solution is

$$\boldsymbol{u}_j = \boldsymbol{K}_j^{-1} \boldsymbol{f}_j = \boldsymbol{E}_j \boldsymbol{f}_j$$

In time domain the solution becomes

$$\boldsymbol{u}(t) = \sum \boldsymbol{u}_j(t)$$

What can be solved with a FEM model?

The idea is, of course, that the FEM model describes the behavior of the studied device well. Basically FEM models are used to solve the response of the model to the given excitation in time domain or in frequency domain. Also internal responses are solved which makes it possible to calculate also strains and stresses of the device. Often they are the most important information.

In the following slides only displacements are discussed, but also strains or stresses can be used the same way, as well as velocities and accelerations.

Also the subscript j is dropped for clarity although the results are for a one excitation frequency.

Solution by excitation forces

The full matrix solution of displacements is

$$u = K^{-1} f = E f$$

E is flexibility matrix. f is the excitation vector which usually has only small number of non-zero degrees of freedom, n_f. The set of these degrees of freedom is marked with f.

Usually we are not interested in all displacements, but only in a selected set. This set is marked with p.

Now only a small part of the flexibility matrix is needed to solve the displacements of the selected set.

$$u_p = E_{pf} f_f$$

This is a linear model from f_f to u_p, so the corresponding surrogate model can be defined with linear regression.

Usually the excitation is not known, but analytical or numerical model of the forces must be used!

Solution by displacements of excitation points…

Usually excitation forces are known only as mathematical formulas. However, they can be calculated with the dynamic stiffness matrix.

$$f = K\,u$$

Again excitation vector f contains only a small number of non-zero degrees of freedom.

Stiffness matrix is usually sparse. Each row has a limited number of non-zero elements. By indexing all non-zero elements on rows corresponding the non-zero excitations we get the set e. Now only a small part of stiffness matrix is needed to solve the excitation forces.

$$f_f = K_{fe}\,u_e$$

With them we can solve the displacements of the selected set.

$$u_p = E_{pf}\,K_{fe}\,u_e = N_{pe}\,u_e \qquad n_e \geq n_f$$

This is a linear model from u_e to u_p, so the corresponding surrogate model can be defined with linear regression.

…Solution by displacements of excitation points

When the FEM model is refined, the nodes of set e move towards excitation points. This model in real world corresponds measuring the displacements of excitation points. It is usually difficult if not impossible. In practice we can only try measure displacements as near as possible the excitation points and get approximations for the displacements of the selected set p.

Solution by displacements of selected points

Though a device is usually excited from only a small set of points, the whole device is excited. So displacements of each point/node contain information of the excitation. It seems reasonable to think that by using a set of “well selected” degrees of freedom (set q) displacements of another selected set can be at least approximated.

$$\boldsymbol{u}_q = \boldsymbol{E}_{qf} \boldsymbol{f}_f \qquad n_q \geq n_f$$

The formal solution is

$$\boldsymbol{f}_f = \left(\boldsymbol{E}_{qf}\right)^{-1} \boldsymbol{u}_q$$

E_{qf} is not a square matrix, so a solution method like QR-method must be used.

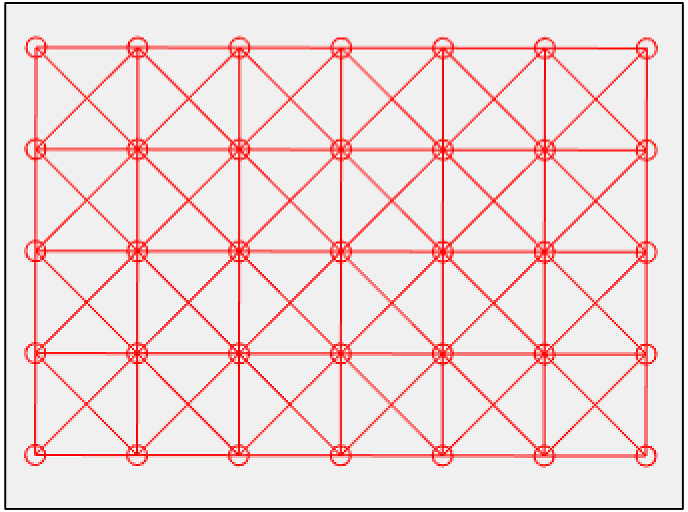
Now the selected set p can be solved.

$$\boldsymbol{u}_p = \boldsymbol{E}_{pf} \left(\boldsymbol{E}_{qf}\right)^{-1} \boldsymbol{u}_q = \boldsymbol{N}_{pq} \boldsymbol{u}_q \qquad n_q \geq n_f$$

This is a linear model from u_q to u_p, so again the corresponding surrogate model can be defined with linear regression.

Surrogate model creation and testing…

The surrogate models developed here were tested using a small FEM model consisting of point masses and springs. Due to the time constraint only a couple of excitation frequencies were tested.



Creation

- A set of 500 loading cases were generated to teach the surrogate models
- Ordinary least squared method was used to create the weight matrix

Testing

- 4 predetermined load cases were used to test the surrogate models

…Surrogate model creation and testing

- The excitation nodes in the test cases were 2nd and 3rd node on the top line (set f)
- The selected nodes were the top corner nodes (set p)
- The (set e) contained all degrees of freedom of elements connected to excitation nodes
- The whole mid row of nodes was used as (set q)

Results

- Solution by excitation forces worked as could be expected
- Solution by displacements of excitation points and of selected points
- Matlab warned of too big condition number for both models when calculating the weights
- All models gave in test cases very accurate results (relative ~ 10^-13)

Conclusions

A general method to create surrogate models for periodically excited devices was developed. The method uses “deformation slices” at selected (angular) frequency points. That is the measured data is transformed to Fourier series by the base period and for every selected frequency a surrogate model is created. The results of the surrogate models can be used directly or they can be combined to recreate time series.

The surrogate model is not restricted to displacements only, but velocities, accelerations, forces, strains, and stresses can be used as well and also mix together.

For linear device/structure the surrogate model can be created using linear regression.

The small FEM model test showed that the developed surrogate model works fast and accurately in right conditions. More research is needed to learn how well it works in practice. Especially how many and to which locations the reference sensors should be placed.